qmm4man (رمز الفصل الالكتروني)

References **1- Theory of Vibration with Applications** **by Thomson**

2- Fundamentals of Vibrations **by Leonard Meirovitch**

Basic Concepts of Vibration

Vibration

Any motion that repeats itself after an interval of time is called vibration

Example

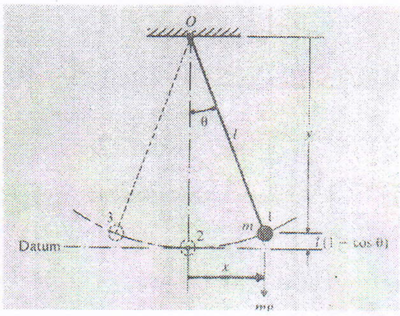
1- Swinging of pendulum
2- A plucked string

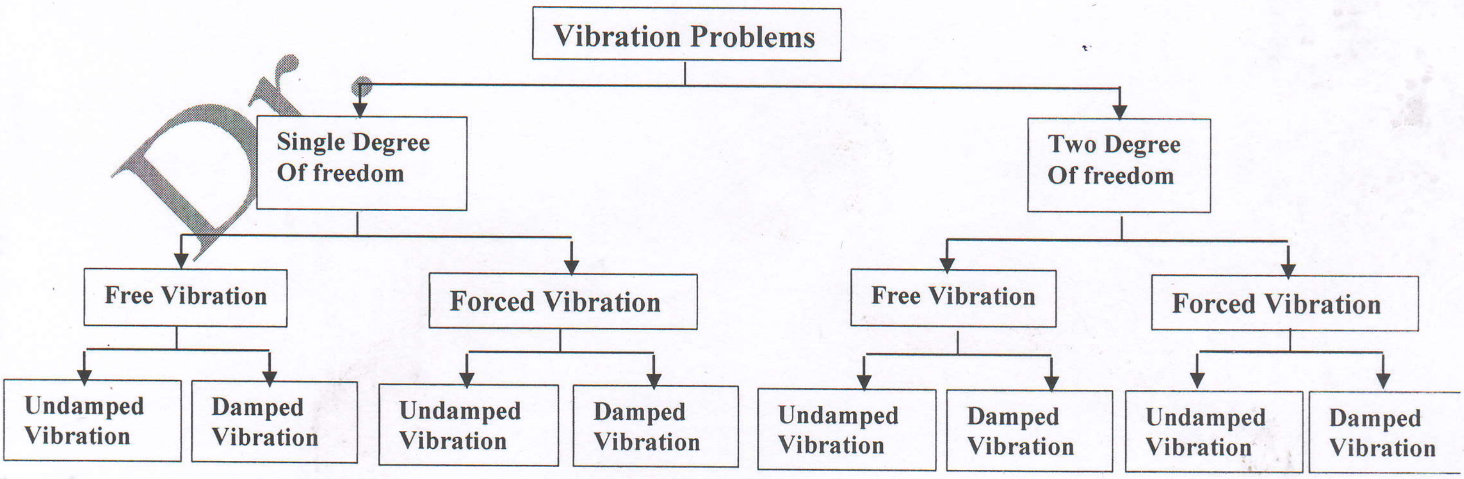
Elementary Parts of Vibrating Systems

A vibratory system, in general includes

1- Means for storing potential energy (spring or elasticity)
2- Means for storing kinetic energy (mass or inertia)
3- Means for which energy is gradually lost (damper)

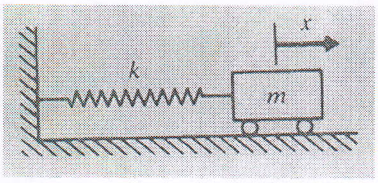
The vibration of a system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternately. If the system is damped, some energy is dissipate id in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

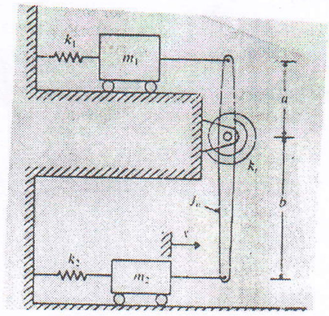




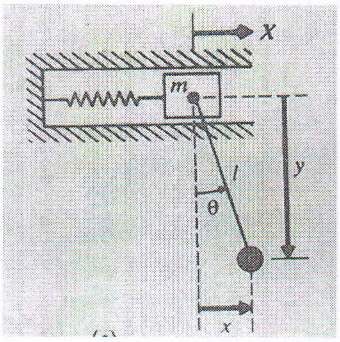
Degree of Freedom

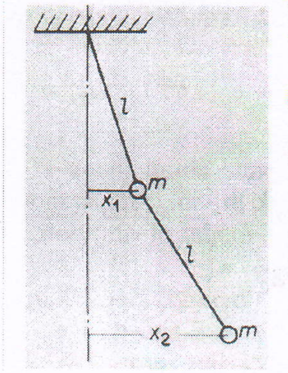
Degree of freedom is the minimum number of <u>independent coordinates</u> required to determine completely the positions of all parts of a system at any instant of time.

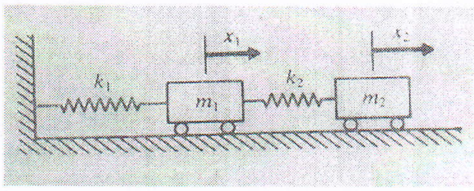




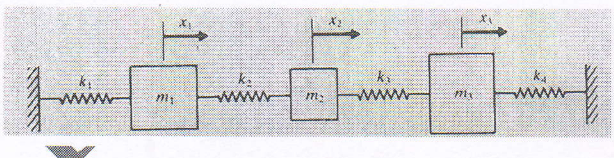
One Degree of freedom

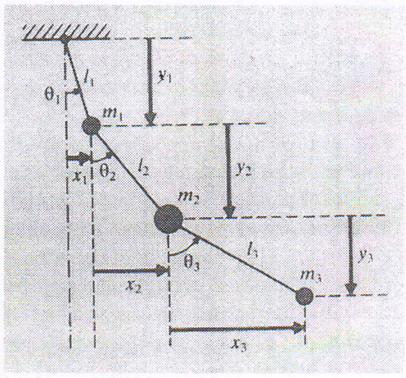






Two Degree of freedom



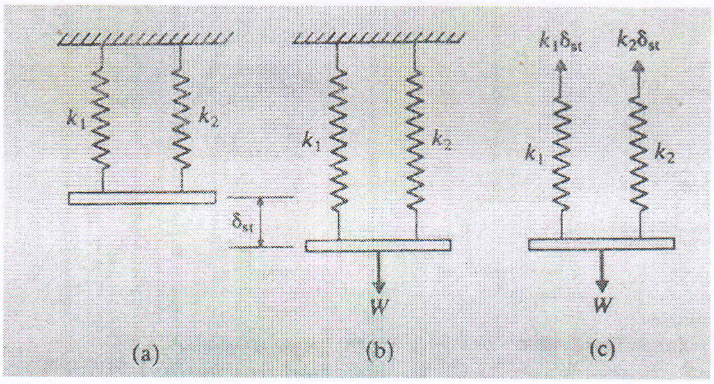


Multi Degree of freedom

Spring force = spring constant (k) × deformation of spring (δ)

Combination of Springs

1- Springs in parallel



In the equilibrium position $\sum \mathbf{F} = 0$

$$W = mg = k_1\delta + k_2\delta \qquad \cdots\cdots\cdots (1)$$

where

k_1, k_2 spring constants (stiffness) (N/m)

δ:- is static deflection of springs

k_{eq} the equivalent spring constant of combination two springs

For the same static deflection δ

$$W = mg = k_{eq}\delta \qquad \cdots\cdots\cdots (2)$$

From equations (1) and (2)

$$\boxed{k_{eq} = k_1 + k_2}$$

For n springs in parallel

$$\boxed{k_{eq} = k_1 + k_2 + k_3 + \cdots + k_n}$$

2- Springs in series

$$\left.\begin{aligned} W &= k_1\delta_1 \\ W &= k_2\delta_2 \end{aligned}\right\} \quad \cdots\cdots\cdots (1)$$

where δ_1 and δ_2 are the elongation of springs 1 and 2 respectively

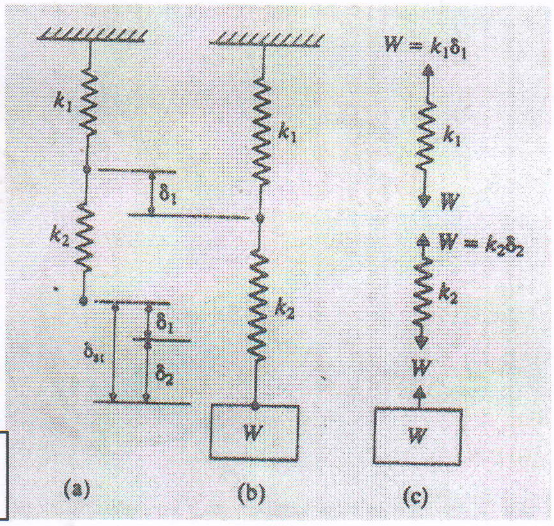
then, the total elongation= $\delta_1 + \delta_2 = \delta_{eq} \quad \cdots (2)$

for combination

$$W = \delta_{eq} \times k_{eq} \quad \cdots\cdots\cdots (3)$$

From equations (1) , (2) and (3)

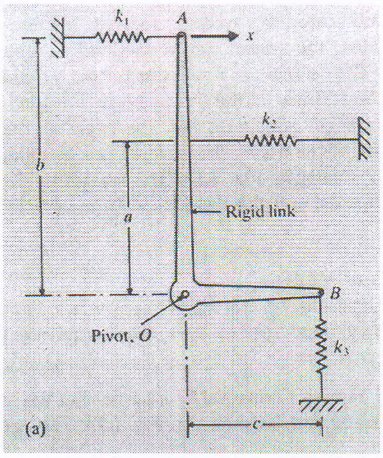
$$\frac{W}{k_{eq}} = \frac{W}{k_1} + \frac{W}{k_2} \Rightarrow \boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$



For n springs in series

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}}$$

3- Springs are connected to levers or gears as shown in Figure below



To find the equivalent spring located in specific point (for example point A) we use potential energy

$x = x_1 = a\sin\theta \quad , \quad x_2 = b\sin\theta$

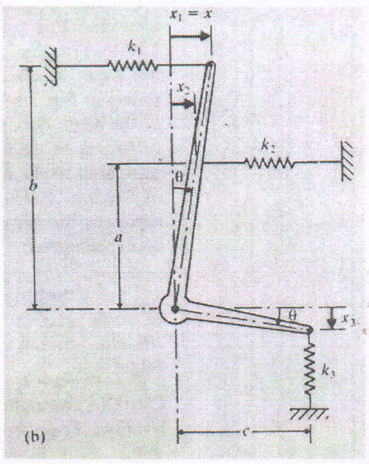
$x_3 = c\sin\theta$

for θ very small $\sin\theta \cong \theta \quad \cos\theta \to 1$

then

$x = a\theta \quad x_2 = b\theta \quad x_3 = c\theta$

$\theta = \frac{x}{a} \quad x_2 = b\frac{x}{a} \quad x_3 = c\frac{x}{a}$



then, the total potential energy stored in three springs is

$$U = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}k_3x_3^2$$

or

$$U = \frac{1}{2}x^2\left[k_1 + \frac{b^2}{a^2}k_2 + \frac{c^2}{a^2}k_3\right]$$

the potential energy stored in spring located at point $A = \frac{1}{2}k_{eq}x^2$

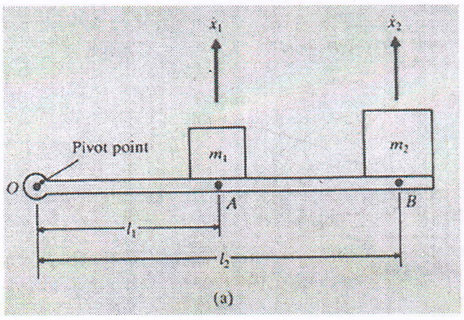
then, equate the two potential energies

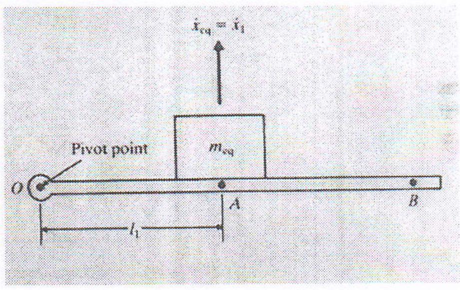
$$\frac{1}{2}k_{eq}x^2 = \frac{1}{2}x^2\left[k_1 + \frac{b^2}{a^2}k_2 + \frac{c^2}{a^2}k_3\right]$$

$$k_{eq} = k_1 + \frac{b^2}{a^2}k_2 + \frac{c^2}{a^2}k_3$$

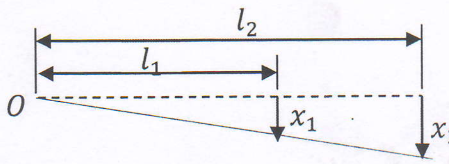
Combination of Masses

To find the equivalent mass located in specific point (for example point A) we use kinetic energy





The total kinetic energy is $\frac{1}{2}m_{eq}\dot{x}_1^2 = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$



but $\quad \frac{x_1}{l_1} = \frac{x_2}{l_2} \qquad x_1 = \frac{l_1}{l_2}x_2 \qquad \dot{x}_2 = \frac{l_2}{l_1}\dot{x}_1$

$$\frac{1}{2}m_{eq}\dot{x}_1^2 = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\left(\frac{l_2}{l_1}\right)^2\dot{x}_1^2$$

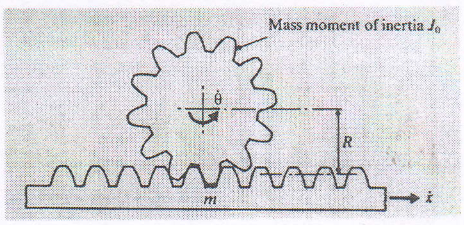
$$m_{eq} = m_1 + m_2\left(\frac{l_2}{l_1}\right)^2$$

Translational and Rotational Masses Coupled Together

let a mass m having a translational velocity $\dot{x}$ be coupled to another mass (of mass moment of inertia J_0) having a rotational velocity $\dot{\theta}$, to find:

equivalent translation mass

the total kinetic energy of the system



$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 \qquad \cdots\cdots\cdots (1)$$

since $\quad x = R\theta \qquad$ then $\qquad \dot{x} = R\dot{\theta}$

then, from Equation (1)

$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\left(\frac{\dot{x}}{R}\right)^2$$

$$m_{eq} = m + \frac{J_0}{R^2}$$

equivalent rotation mass

the total kinetic energy of the system

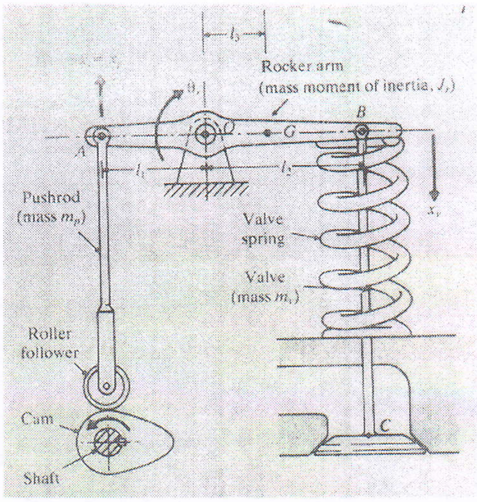
$$\frac{1}{2}J_{eq}\dot{\theta}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 \qquad \text{since} \quad x = R\theta \qquad \text{then} \quad \dot{x} = R\dot{\theta}$$

so $$\frac{1}{2}J_{eq}\dot{\theta}^2 = \frac{1}{2}m(R\dot{\theta})^2 + \frac{1}{2}J_0\dot{\theta}^2$$

$$J_{eq} = J_0 + mR^2$$

Example

A cam – follower mechanism shown in Figure is used to convert the rotary motion of a shaft into the oscillating or reciprocating motion of a valve. The follower system consists of a push rod, a rocker arm, a valve, and a valve spring. Find the equivalent mass (m_{eq}) of this cam – follower system by assuming the locating (m_{eq}) of as point A.

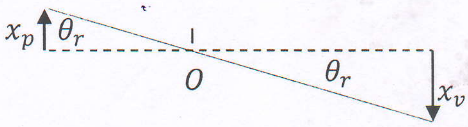


Solution

the total kinetic energy of the system

$$= \frac{1}{2}m_p\dot{x}_p^2 + \frac{1}{2}m_v\dot{x}_v^2 + \frac{1}{2}J_r\dot{\theta}_r^2 \qquad \cdots\cdots\cdots (1)$$

If m_{eq} denotes the equivalent mass placed at point A, then



$$\dot{x}_{eq} = \dot{x}$$

so, $$T_{eq} = \frac{1}{2}m_{eq}\dot{x}^2 \qquad \cdots\cdots\cdots (2)$$

$$\sin\theta_r = \frac{x}{l_1} = \frac{x_v}{l_2} \qquad x_v = \frac{l_2}{l_1}x \qquad \dot{x}_v = \frac{l_2}{l_1}\dot{x}$$

and $$\theta_r = \frac{\dot{x}}{l_1}$$

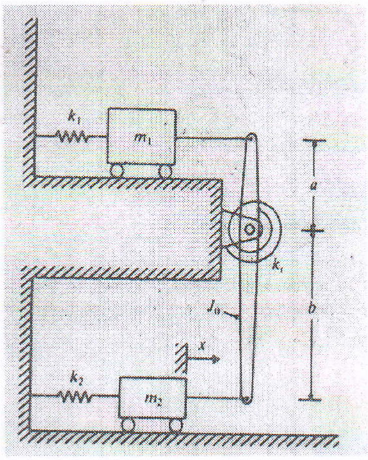
then, from equations (1) and (2)

$$\frac{1}{2}m_{eq}\dot{x}^2 = \frac{1}{2}m_p\dot{x}_p^2 + \frac{1}{2}m_v\left(\frac{l_2}{l_1}\dot{x}\right)^2 + \frac{1}{2}J_r\left(\frac{\dot{x}}{l_1}\right)^2$$

$$m_{eq} = m_p + m_v \left(\frac{l_2}{l_1}\right)^2 + \frac{J_r}{l_1^2}$$

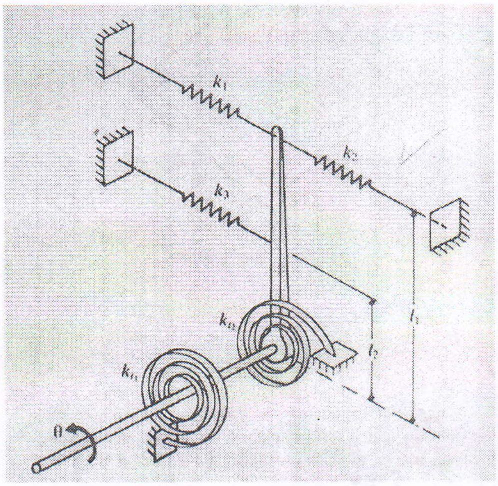
HW 1

Find the equivalent mass of the rocker arm assembly, referred to the x coordinate.



HW 2

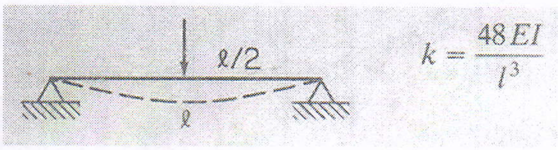
In the Figure find the equivalent spring constant of the system in the direction θ



Equivalent Spring Constants

1- Simple Supported Beam

From strength of materials



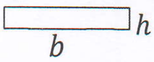
$$\delta = \frac{Wl^3}{48\,EI} \qquad \cdots\cdots\cdots (1)$$

where

EI:- bending stiffness $(N.m^2)$

E:- modulus of elasticity (N/m^2)

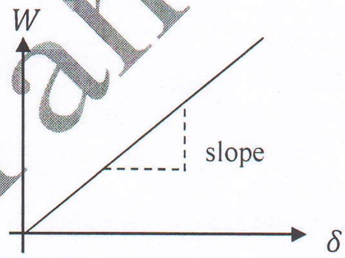
I:- second moment of area (m^4)



$I = \frac{bh^3}{12}$

$I = \frac{\pi d^4}{64}$

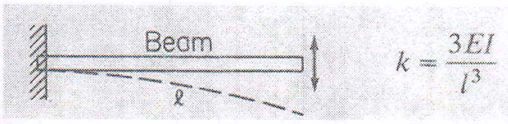
W:- applied load



Then from Equation (1) $\boxed{k = \frac{W}{\delta} = \frac{48\,EI}{l^3}}$ = slope of $W - \delta$ curve

2- Cantilever Beam

Similarly From strength of materials

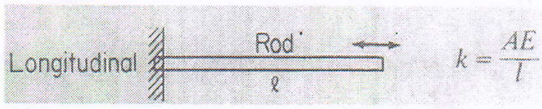


$$\delta = \frac{Wl^3}{3\,EI} \qquad \cdots\cdots\cdots (1)$$

$$\boxed{k = \frac{W}{\delta} = \frac{3\,EI}{l^3}}$$

3- Rod or Cable (Longitudinal or Axial Deformation)

From strength of materials



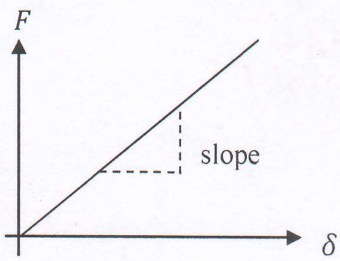
$$\delta = \frac{Fl}{EA} \qquad \cdots\cdots\cdots (1)$$

where

E:- modulus of elasticity (N/m^2)

A:- Cross sectional area (m^2)

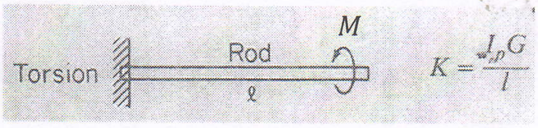
F:- applied load



Then from Equation (1) $\boxed{k = \frac{W}{\delta} = \frac{EA}{l}}$ = slope of $W - \delta$ curve

4- Shaft in Torsion

From strength of materials



$$M = \frac{GJ}{l}\theta \qquad \cdots\cdots\cdots (1)$$

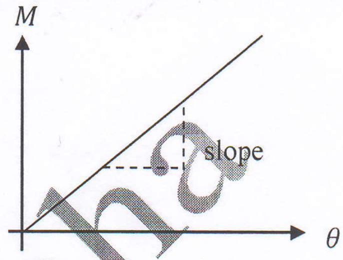
where

G:- modulus of rigidity (N/m^2)

J:- second polar moment of area $(m^4) = \frac{\pi d^4}{32}$

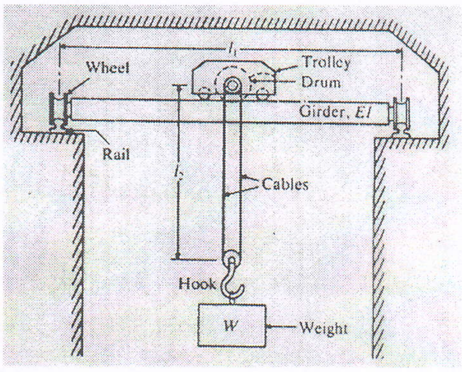
M:- applied torque

Then from Equation (1) $\boxed{k = \frac{M}{\theta} = \frac{GJ}{l}}$ = slope of $M - \theta$ curve



Example

A weight W is being lifted by an overhead traveling crane as shown in Figure. The girder is a uniform beam of length l_1 and flexural rigidity EI and each of the two cables has a length l_2, diameter d, and young modulus E. Assuming the masses of the trolley, motor, drum, cables and hooks to be negligible, find the equivalent spring constant of the girder and the cable.

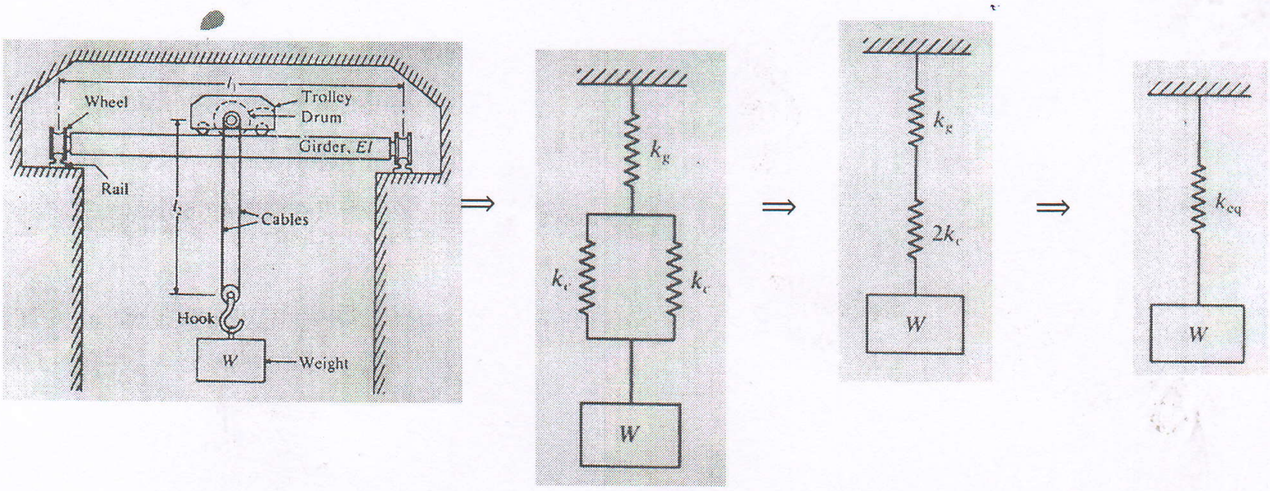


Solution

The girder is assumed to be simply supported beam with central load

From table $k_g = \frac{48\, E_g I}{l_1^3}$ where E_g = modulus of elasticity of girder

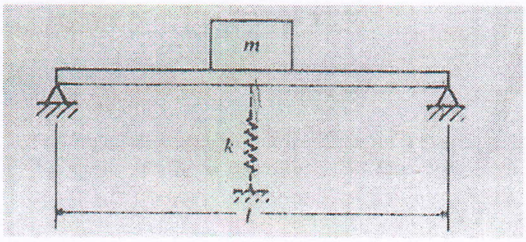
since, the cables subjected to axial load , then $k_c = \frac{A\, E_c}{l_2}$ where E_c = modulus of elasticity of cable



$$\frac{1}{k_{eq}} = \frac{1}{k_g} + \frac{1}{2k_c} \qquad \Rightarrow \qquad k_{eq} = \frac{2k_c k_g}{k_g + 2k_c}$$

H.W3

A machine of mass $m = 500$ kg is mounted on a simply supported steel beam of $l = 2$ m having rectangular cross section (depth = 0.1 m, width = 1.2 m) and Young's modulus = 2.06×10^{11} N/m^2. To reduce the vertical deflection of the beam, a spring of stiffness k is attached at the mid-span, as shown in Figure. Determine the value of k needed to reduce the deflection of the beam to one-third of its original value. Assume that the mass of the beam is negligible.



Single Degree of Freedom

Equation of Motion: Natural Frequency

Let m mass of single degree of freedom system shown in Figure (kg)

Δ : the static deformation of the spring from the static equilibrium position (m)

k: be the stiffness of the spring (N/m)

then, $k\Delta = W = mg$ ……… (1)

pull the mass down ward and released it (**free vibration**) then apply the Newton's second law at any position unless equilibrium position

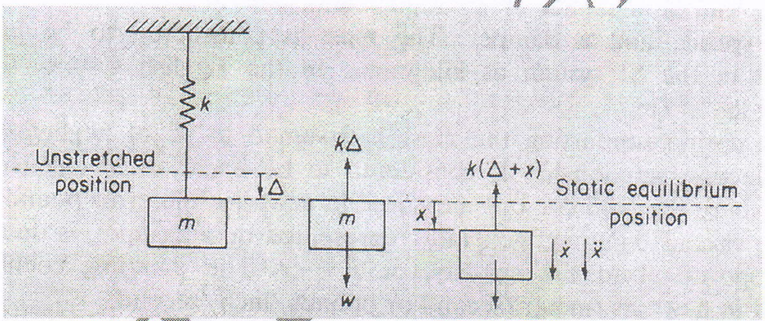
$\sum \mathbf{F} = m\ddot{x}$ where $\ddot{x}$ acceleration of the mass

Then, **From Free Body Diagram**

$$m\ddot{x} = W - k(\Delta + x)$$

From Equation (1)

$$m\ddot{x} = -kx \qquad \text{or} \qquad m\ddot{x} + kx = 0$$



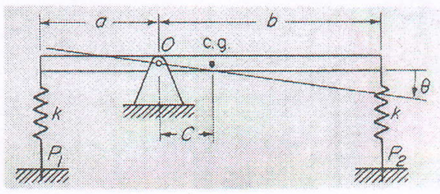
By defining the circular frequency $\omega_n^2 = \frac{k}{m}$

where ω_n is the natural frequency of the system (rad/s)

$$\ddot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = 0 \quad \Rightarrow \quad \boxed{\ddot{x} + \omega_n^2 x = 0} \text{ (Equation of Motion)}$$

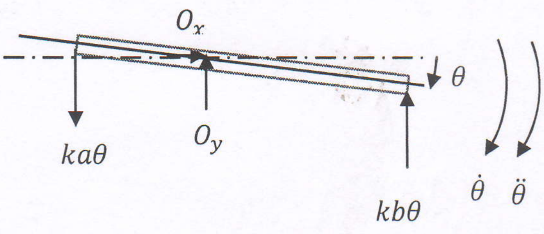
Example

Figure shows a uniform bar pivoted about point O with springs of equal stiffness k at each end. The bar is horizontal in the equilibrium position. Determine the equation of motion and its natural frequency.



Solution

Give the bar an rotational motion θ as shown



$\sum M_0 = J_0\ddot{\theta}$ where $\ddot{\theta}$ is angular acceleration

then, **From Free Body Diagram**

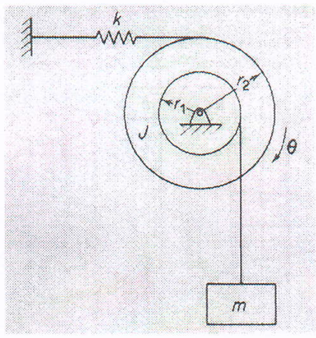
$$(-ka\theta)a - (-kb\theta)b = J_0\ddot{\theta} \qquad \text{or} \qquad J_0\ddot{\theta} + (ka^2 + kb^2)\theta = 0$$

$$\ddot{\theta} + \frac{k(a^2+b^2)}{J_0}\theta = 0$$ **(Equation of Motion)**

and $$\omega_n = \sqrt{\frac{k(a^2+b^2)}{J_0}}$$

Example

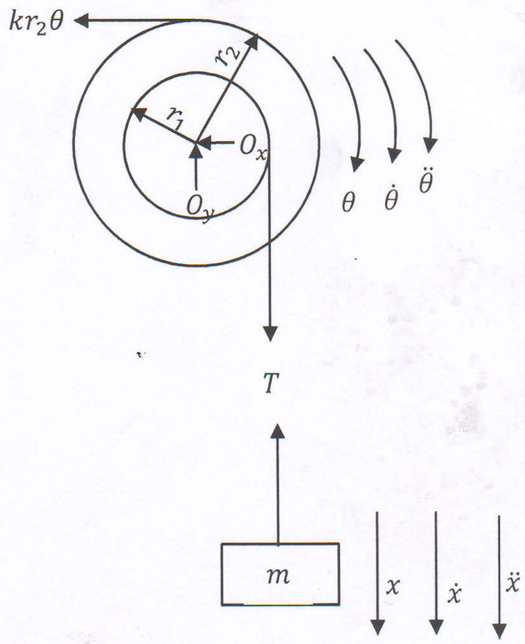
Find the equation of motion and natural frequency of the system shown in Figure. The moment of inertia of combined disc about O is J.



Solution

give the combined disc an rotational motion θ as shown

apply the Newton's second law to mass (m)



$\sum \mathbf{F} = m\ddot{x}$ where $\ddot{x}$ acceleration of the mass

Then, **From Free Body Diagram**

$$m\ddot{x} = -T \qquad \cdots\cdots\cdots (1)$$

then apply the Newton's second law to disk (J)

$\sum M_0 = J\ddot{\theta}$ where $\ddot{\theta}$ is angular acceleration

then, **From Free Body Diagram**

$$(-kr_2\theta)r_2 + Tr_1 = J\ddot{\theta} \qquad \cdots\cdots\cdots (2)$$

but $x = r_1\theta$ $\qquad \ddot{x} = r_1\ddot{\theta}$

from Equation (1) $\qquad T = -mr_1\ddot{\theta} \qquad \cdots\cdots\cdots (3)$

substitution (3) into (2)

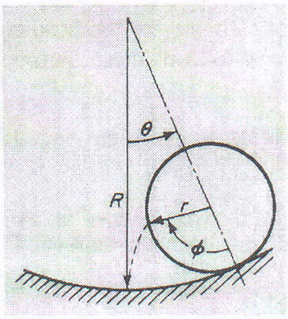
$$-kr_2^2\theta - mr_1^2\ddot{\theta} = J\ddot{\theta}$$

$$(J + mr_1^2)\ddot{\theta} + kr_2^2\theta \qquad \qquad \ddot{\theta} + \frac{kr_2^2}{J+mr_1^2}\theta = 0$$ **(Equation of Motion)**

and $\omega_n = \sqrt{\dfrac{kr_2^2}{J+mr_1^2}}$

Example

A uniform rigid disk of radius r rolls without slipping inside a circular track of radius R, as shown in Figure. For small oscillation **find** the equation of motion and natural frequency of oscillation. ($m =$ mass of disk, $J =$ mass moment of inertia of disk about its center)



Solution

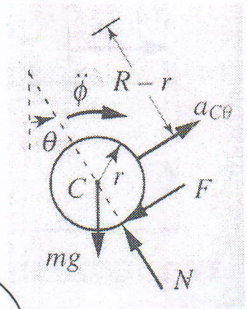
give the disc an motion s shown (Note that the disk has two motions 1- rotation 2- translation)

apply the Newton's second law to mass (m)

$\sum F_\theta = m\, a_{c\theta}$ where $a_{c\theta}$ acceleration of the mass center in the θ-direction

then, **From Free Body Diagram**

$$F - mg\sin\theta = m(R-r)\ddot{\theta} \quad \cdots\cdots (1)$$



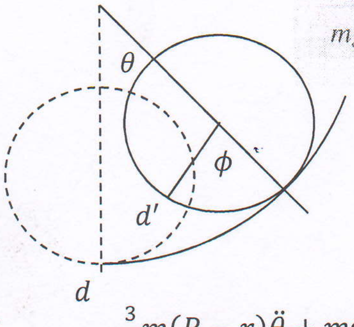
for rotation motion

$\sum M_C = J\ddot{\phi}$ ϕ: − is pure rotation of the disk

$Fr = \dfrac{mr^2}{2}\ddot{\phi}$ $\cdots\cdots (2)$ $J = \dfrac{mr^2}{2}$

from the geometry of the problem

$(R-r)\theta = r\phi \quad \Rightarrow \quad (R-r)\ddot{\theta} = r\ddot{\phi}$



from Equation (2)

$$F = \frac{mr}{2}\frac{(R-r)}{r}\ddot{\theta}$$

substitution into (1), yield

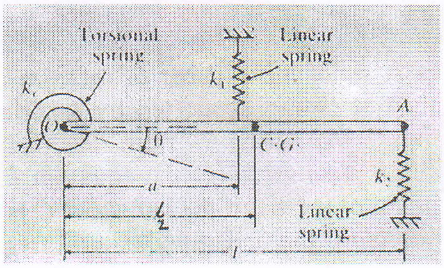
$$-\frac{m}{2}(R-r)\ddot{\theta} - mg\sin\theta = m(R-r)\ddot{\theta} \quad \Rightarrow \quad \frac{3}{2}m(R-r)\ddot{\theta} + mg\sin\theta = 0$$

For small angle θ $\sin\theta \cong \theta$ and $\cos\theta \cong 1$

$\ddot{\theta} + \dfrac{2g}{3(R-r)}\theta = 0$ **(Equation of Motion)** $\omega_n = \sqrt{\dfrac{2g}{3(R-r)}}$ **(Natural Frequency)**

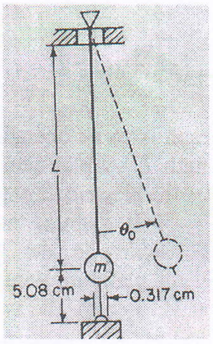
H.W4

Find the equation of motion and natural frequency of the system shown in Figure



H.W5

Find the equation of motion and natural frequency of the systems shown in Figures



Translation motion	Rotational motion
Mass (kg)	Mass moment of inertia ($kg.m^2$)
Spring k (N/m)	Torsional spring k_t (N.m/rad)
Damper C (N.m/s)	Torsional damper C_t (N.m.s/rad)